

## Control Sec 5

### \* Frequency Domain Analysis

steady state Response to sinusoidal input

$$r(t) = A \sin(\omega t)$$

$$\begin{aligned} & \left\{ \begin{aligned} Y(s) &= T(s) R(s) \\ \rightarrow R(s) &= \frac{A}{s^2 + \omega^2} \end{aligned} \right. \end{aligned}$$

$$Y(s) = T(s) \frac{A}{s^2 + \omega^2}$$

Assume:

$$T(s) = \frac{N(s)}{\prod_i (s + p_i)}$$

$$Y(s) = \frac{A N(s)}{(s^2 + \omega^2) \prod_i (s + p_i)}$$

$$y(t) = \mathcal{L}^{-1} [Y(s)]$$

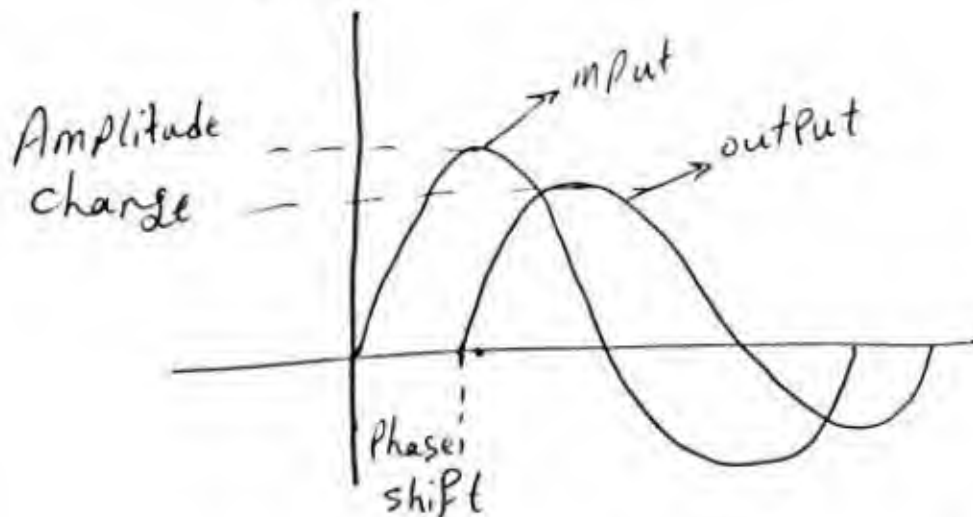
$$Y(s) = \frac{C_1}{s+p_1} + \frac{C}{s+p_2} + \dots + \frac{\alpha s + B}{s^2 + \omega^2}$$

$$y(t) = C_1 e^{-p_1 t} + C_2 e^{-p_2 t} + \dots + \alpha \cos(\omega t) + \frac{B}{\omega} \sin \omega t$$

$$y(t) = C_1 e^{-p_1 t} + C_2 e^{-p_2 t} + \dots + C \sin(\omega t - \phi)$$

→ at steady state

$$y(t) \Big|_{t=\infty} = C \sin(\omega t - \phi)$$



input was  $y(t) = A \sin(\omega t)$

→ Magnitude change & Phase shift  $\Rightarrow$  depend on Frequency response of the system (Bandwidth)

\* Fourier Transform

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

From Laplace Transform

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt$$

$s \rightarrow j\omega \Rightarrow \text{Laplace} \rightarrow \text{Fourier}$

For given system represented by T.F  $T(s)$  replace of every  $s$  by  $j\omega \rightarrow$  The system will be described in Frequency domain.

$$\text{T.F : } G \cdot H(j\omega) = \underbrace{|G H(j\omega)|}_{\substack{\text{Magnitude} \\ \text{ser 5 } [3]}} \angle \underbrace{G H(j\omega)}_{\text{Phase shift}}$$

## \* Frequency domain Analysis

① Polar Plots.

② Bode diagram

"Bode diagram"

$$|GH(j\omega)| = \frac{\prod_i |(j\omega + z_i)|}{\prod_k (j\omega + p_k)}$$

$$\log |GH(j\omega)| = \log |(j\omega + z_1)| + \log |(j\omega + z_2)| + \dots - \log |(j\omega + p_1)| - \log |(j\omega + p_2)| + \dots$$

→ Bode diagram

$$\rightarrow \log |GH(j\omega)|$$

$\downarrow$   
 $L_m$

v.s

$\omega$   
(log)

→

$\phi$

v.s

$\omega$

$$* s^{\pm n}$$

$$L_m = \pm 20n \log(w)$$

معادله فرم ال  $L_m$  و.س  $\log(w)$  خط مستقیم  
 و دد علی ال (semi log)

$$\phi = \pm 90n$$

$$* \underline{K s^{\pm n}}$$

$$\rightarrow L_m = 20 \log(K) \pm 20n \log(w)$$

$$\rightarrow \phi = \pm 90n \quad \underline{\text{cause}} \text{ phase of } K = 0$$

$$* (1 + Ts)^{\pm n}$$

$$w \ll \frac{1}{T} \Rightarrow L_m = 0$$

$$w \gg \frac{1}{T} \Rightarrow L_m = \pm 20 \log(wT)$$

$$\phi = \pm n \tan^{-1}(wT)$$

Ex

$$GH(s) = \frac{K}{s(s+2)(s+10)}$$

K such that  $e_{ss} \leq 0.1$  for unit ramp

Sol

→ For unit ramp:

$$e_{ss} = \frac{1}{K_v}, \quad K_v = \lim_{s \rightarrow 0} s GH(s)$$

$$K_v = \frac{1}{e_{ss}} = 10 \Rightarrow 10 = \lim_{s \rightarrow 0} \frac{sK}{s(s+2)(s+10)}$$

$$\therefore K = 200$$

$$\therefore GH(s) = \frac{200}{s(s+2)(s+10)}$$

① system → Time Constant form

$$GH(s) = \frac{200^{10}}{10 \times 2 \times s(1+0.5s)(0.1s+1)}$$

$$\underline{s \rightarrow j\omega}$$

$$GH(j\omega) = \frac{10}{j\omega (0.5j\omega + 1) (0.1j\omega + 1)}$$

corner Frequency

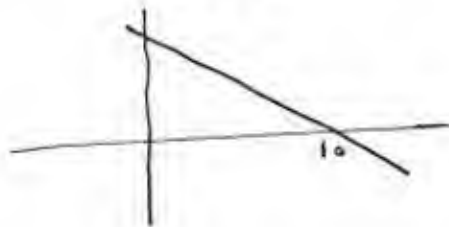
$$\omega_{c1} = 2$$

$$\omega_{c2} = 10$$

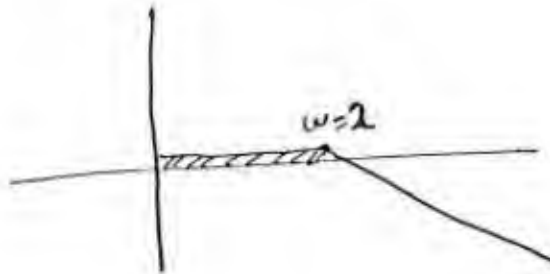
term

$$\frac{10}{j\omega}$$

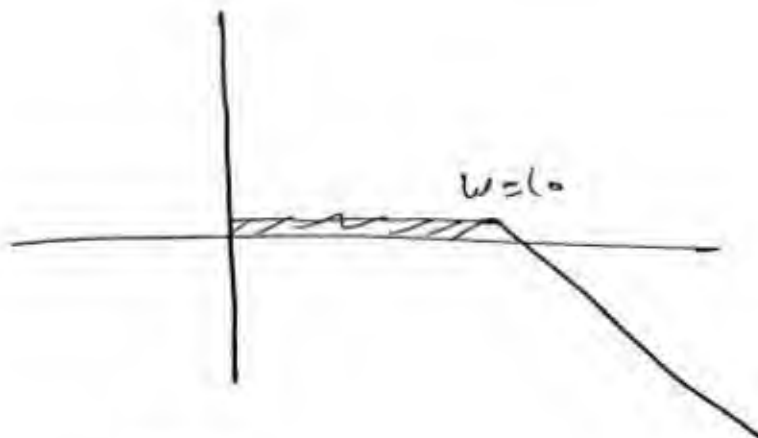
Curve

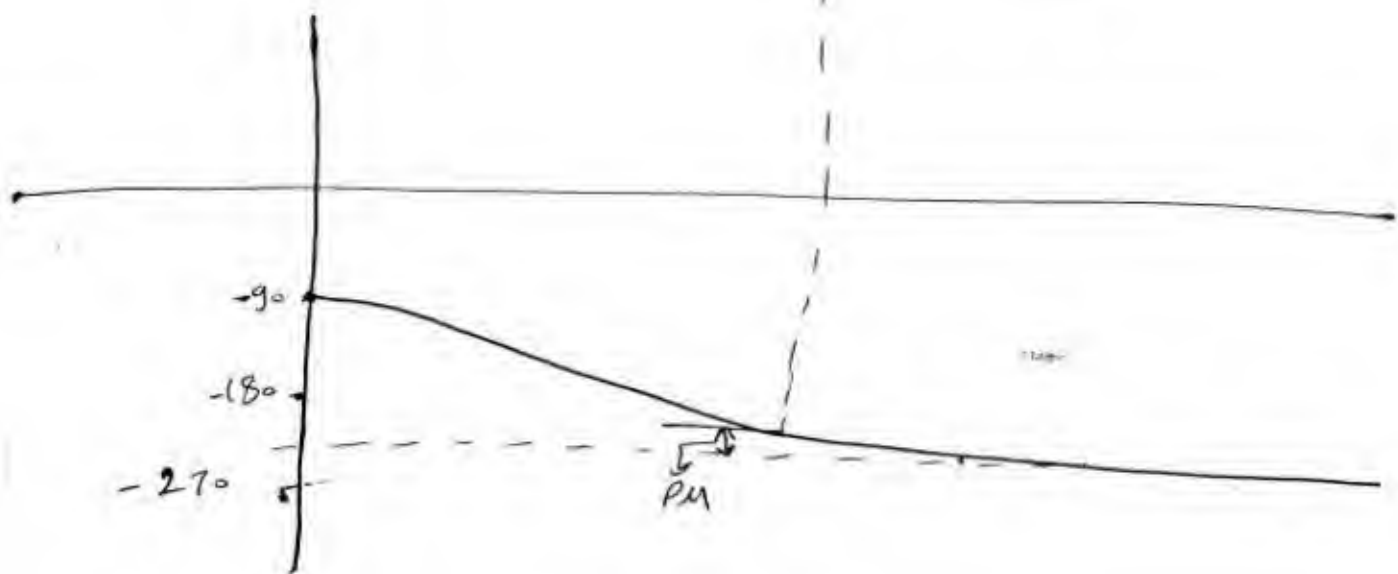
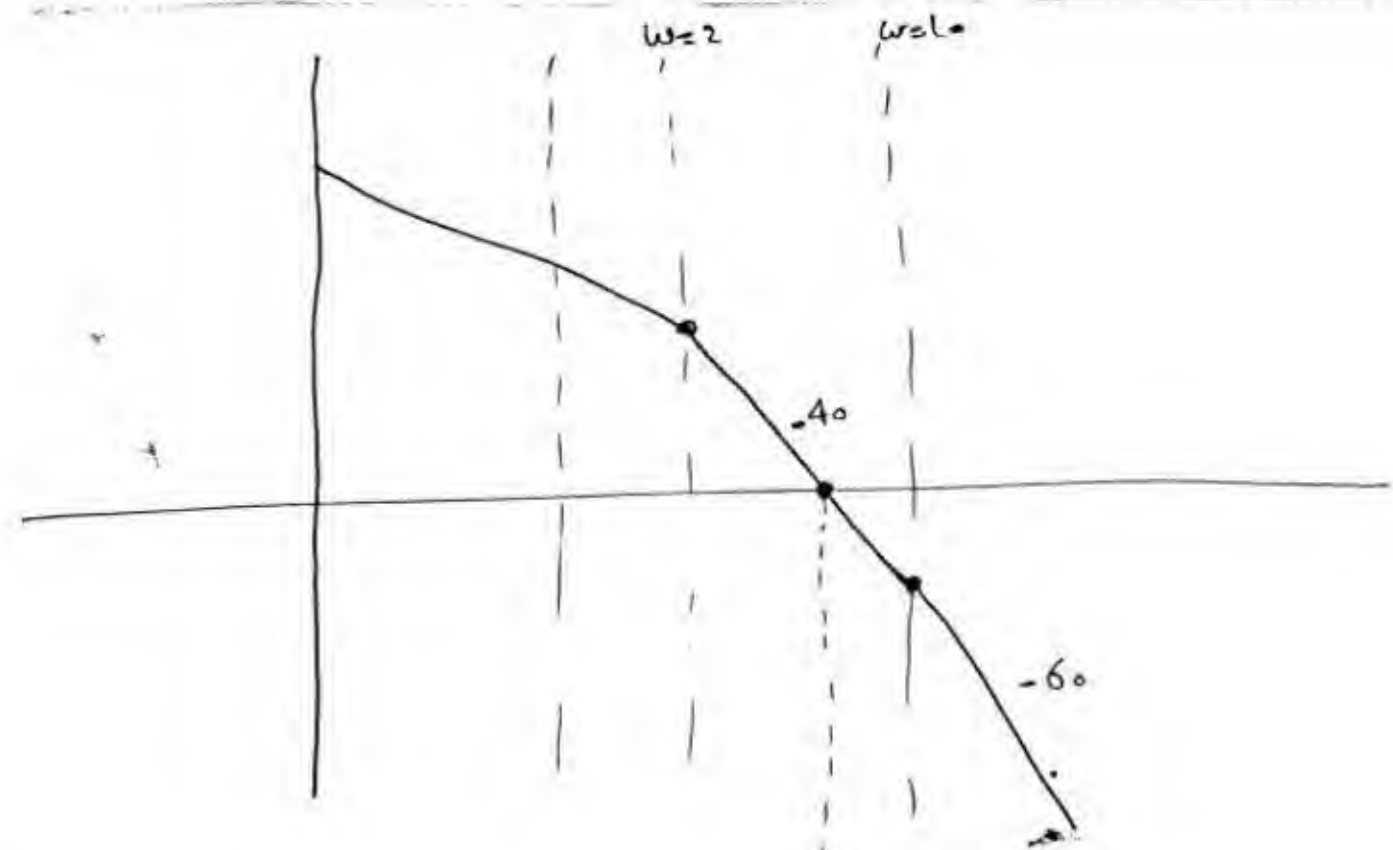


$$\frac{1}{j0.5\omega + 1}$$



$$\frac{1}{j0.1\omega + 1}$$





$$\phi = -90 - \tan^{-1} 0.5 - \tan^{-1} 0.1w$$

$w$	0	1	2	10	$\infty$
$\phi$	-90	-122.3	-146.3	-213.9	-270

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Sec 5



GM

$$GM = \frac{1}{M} \Big|_{\phi = -180} \text{ at } \omega_{pc}$$

$$20 \log GM = 20 \log \frac{1}{M}$$

$$GM|_{dB} = -20 \log M = -L_m$$

GM  $\begin{cases} > 1 & \text{stable} \\ = 1 & \text{critically} \\ < 1 & \text{unstable} \end{cases}$

GM|<sub>dB</sub>  $\begin{cases} > 0 & \text{stable} \\ = 0 & \text{critically} \\ < 0 & \text{unstable} \end{cases}$

$$GM = 0 \quad \omega_{pc} = 4.5$$

$$PM = 180 + \phi \Big|_{M=1} \quad @ \omega_{gc}$$
$$L_m = 0$$

$$\omega_{gc} = 4.5$$

$$PM = 0$$

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